

# Cosmological perturbation and matter power spectrum in bimetric massive gravity

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## Abstract

We study the bimetric massive gravity theory. By concentrating on the minimal scenario, we discuss the linear perturbation equations in the synchronous gauge. We find that the effective dark energy equation of state always stays in the phantom phase, and the matter power spectrum is suppressed. In addition, we discuss the ghost and stability problems and show that the allowed window for the model parameter is constrained to be very small by the large scale structure observations.

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## I. INTRODUCTION

As the cosmological observations [1–5] have shown that our universe is undergoing an accelerated cosmic expansion, the Einstein equation in general relativity (GR) without the mystery cosmological constant term needs to be modified. Among these modified gravity theories, the DGP braneworld [6] and massive gravity theories can both explain the late-time acceleration without adding dark energy additionally.

Massive gravity with a massive spin-2 field was first constructed by Fierz and Pauli [7] in 1939. However, with the van Dam-veltman-Zakharov discontinuity [8–10], the Fierz-Pauli’s theory predicts the bending of light around the sun, which mismatches the current solar system observations. Although Vainshtein [11] proposed a mechanism of nonlinear interactions, which could recover GR in the zero graviton mass limit, it suffers from the Boulware Deser (BD) ghost instability at the same time [12].

In recent years, de Rham, Gabadadze and Tolley (dRGT) [13, 14] have successfully built the covariant theory of massive gravity, which can be ghost-free in the decoupling limit to both linear and non-linear orders by introducing a second reference metric in addition to the ordinary one. Nevertheless, the homogeneous and isotropic cosmological solutions are not stable in such a non-linear theory [15–17]. To solve this problem, Hassan and Rosen have extended massive gravity with a non-dynamic second metric to bigravity with a dynamic one [18–20]. Besides being ghost-free for massive gravitons, there exist some solutions in this bimetric theory with interacting massless and massive spin-2 fields. Furthermore, this model can be applicable to realize the late-time accelerated cosmic expansion without a cosmological constant [21, 22]. As the theory contains five free parameters, there are various possible cosmological solutions with different choices of these coupling parameters. However, many bigravity models encounter with Higuchi ghosts [23, 24] or gradient instabilities under cosmological perturbations [25–31]. There are several attempts to find viable cosmological solutions [32–40]. In particular, the general conditions have been given in Ref. [41] to avoid the Higuchi ghosts and gradient instabilities at the linear level. Some recent studies on bigravity can be referred to Refs. [42–58] and references therein.

In this work, we explore the feature of the linear density perturbations as well as matter power spectrum for a non-trivial minimal scenario in the bimetric massive gravitational theory. We will first estimate the allowed window of the model parameter by comparing our

numerical calculations to the observational data, and then examine the stability with the constrained parameter.

This paper is organized as follows. In Sec. II we derive the effective equations of dark energy at the background level. In Sec. III, we consider the scalar perturbations in the synchronous gauge and obtain the perturbed equations. We numerically solve the linear perturbation equations in Sec. IV. We discuss the ghost and stability problems and show the matter power spectrum in Sec. V. Finally, the conclusions are given in Sec. VI.

## II. BACKGROUND OF BIMETRIC MASSIVE GRAVITY

The action of the bimetric massive gravity theory is given by [18],

$$\mathcal{S} = - \int d^4x \left( \frac{\sqrt{-g}}{16\pi G} R(g) + \frac{\sqrt{-f}}{16\pi G_f} R(f) \right) + m^2 \int d^4x \frac{\sqrt{-g}}{8\pi G} \sum_{n=0}^4 \beta_n e_n(\mathbb{X}) + \mathcal{S}_M(g_{\mu\nu}, \Psi), \quad (1)$$

where  $R(g)$  and  $R(f)$  are the Ricci scalars, corresponding to the ordinary and new metrics  $g_{\mu\nu}$  and  $f_{\mu\nu}$ , respectively,  $m^2$  is a mass parameter,  $\mathcal{S}_M(g_{\mu\nu}, \Psi)$  is the action of the matter term with the matter field  $\Psi$ ,  $\beta_n$  are the arbitrary constants, and  $e_n$  are defined by,

$$\begin{aligned} e_0(\mathbb{X}) &= 1, & e_1(\mathbb{X}) &= [\mathbb{X}], & e_2(\mathbb{X}) &= \frac{1}{2}([\mathbb{X}]^2 - [\mathbb{X}^2]), \\ e_3(\mathbb{X}) &= \frac{1}{6}([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3]), & e_4(\mathbb{X}) &= \det \mathbb{X}, \end{aligned} \quad (2)$$

with  $\mathbb{X} = \sqrt{g^{\alpha\beta} f_{\beta\gamma}}$ . For convenience, we will absorb  $m^2$  into  $\beta_n$ , set  $8\pi G_f = 1$  [60, 61], and denote the trace of the matrix  $\mathbb{X}$  by  $[\mathbb{X}]$ .

Varying the action in Eq. (1) with respect to  $g_{\mu\nu}$  and  $f_{\mu\nu}$ , the modified Einstein equations can be derived to be,

$$G_{\mu\nu} + \sum_{n=0}^3 (-1)^n \beta_n g_{\mu\lambda} (J_n)^\lambda_\nu = \kappa^2 T_{\mu\nu}^M, \quad (3)$$

$$F_{\mu\nu} + \sum_{n=0}^3 (-1)^n \beta_{4-n} f_{\mu\lambda} (J_n)^\lambda_\nu = 0, \quad (4)$$

respectively, where  $\kappa^2 = 8\pi G = 1$ ,  $G_{\mu\nu}$  ( $F_{\mu\nu}$ ) is the Einstein tensor for the metric  $g_{\mu\nu}$  ( $f_{\mu\nu}$ ),  $T_{\mu\nu}^M$  is the energy-momentum tensor, and  $(J_n)^\lambda_\nu$  are defined by

$$\begin{aligned} J_0 &= \mathbb{I}, & J_1 &= \mathbb{X} - \mathbb{I}[\mathbb{X}], & J_2 &= \mathbb{X}^2 - \mathbb{X}[\mathbb{X}] + \frac{1}{2}\mathbb{I}([\mathbb{X}]^2 - [\mathbb{X}^2]), \\ J_3 &= \mathbb{X}^3 - \mathbb{X}^2[\mathbb{X}] + \frac{1}{2}\mathbb{X}([\mathbb{X}]^2 - [\mathbb{X}^2]) - \frac{1}{6}\mathbb{I}([\mathbb{X}]^3 - 3[\mathbb{X}][\mathbb{X}^2] + 2[\mathbb{X}^3]). \end{aligned} \quad (5)$$

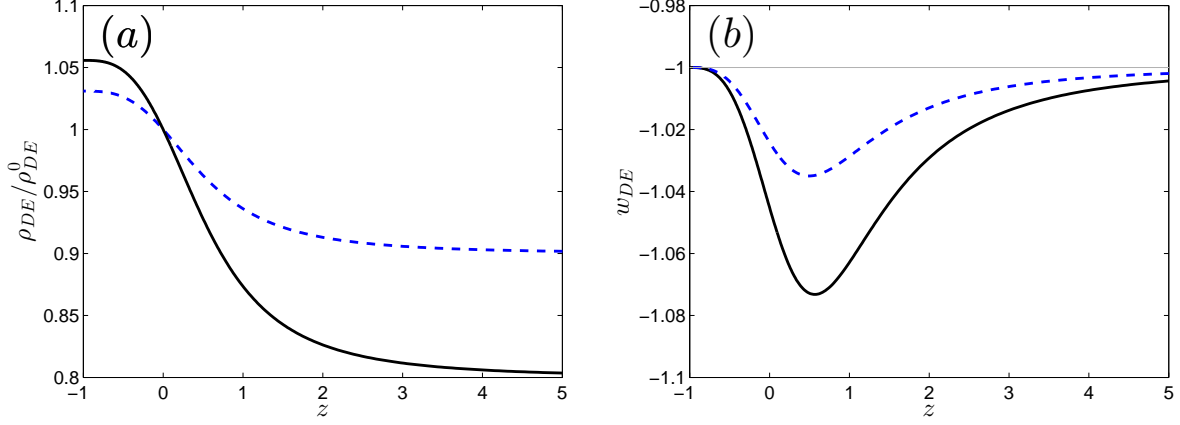


FIG. 1: Evolutions of (a)  $\rho_{DE}$  and (b)  $w_{DE}$  as functions of the redshift  $z$  with  $(\bar{\beta}_0, \bar{\beta}_1) = (0.8, 0.2)$  (black solid line) and  $(0.9, 0.1)$  (blue dashed line), where the boundary conditions of  $\Omega_m = 0.26$  and  $\Omega_r = 8.4 \times 10^{-5}$  are used.

By taking the Friedmann-Lemaître-Robertson-Walker (FLRW) types of the metric [61],

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a(t)^2 dx^i dx_i \quad (6)$$

$$ds_f^2 = f_{\mu\nu} dx^\mu dx^\nu = -\frac{\dot{b}^2}{\dot{a}^2} dt^2 + b(t)^2 dx^i dx_i \quad (7)$$

$$(8)$$

the Friedmann equations are given by

$$H^2 = \frac{1}{3} \left( \rho_M + \beta_0 + 3\beta_1 \frac{b}{a} + 3\beta_2 \frac{b^2}{a^2} + \beta_3 \frac{b^3}{a^3} \right), \quad (9)$$

$$\dot{H} = -\frac{1}{2} \left( \rho_M + P_M + \beta_1 \frac{b}{a} + 2\beta_2 \frac{b^2}{a^2} + \beta_3 b^3 a^3 - \beta_1 \frac{\dot{b}}{\dot{a}} - 2\beta_2 \frac{b}{a} \frac{\dot{b}}{\dot{a}} - \beta_3 \frac{b^2}{a^2} \frac{\dot{b}}{\dot{a}} \right), \quad (10)$$

and

$$H^2 = \frac{1}{3} \left( \beta_1 \frac{a}{b} + 3\beta_2 + 3\beta_3 \frac{b}{a} + \beta_4 \frac{b^2}{a^2} \right), \quad (11)$$

$$H^2 + 2 \frac{H}{H_f} \frac{\ddot{a}}{a} = \left( \beta_2 + 2\beta_3 \frac{b}{a} + \beta_4 \frac{b^2}{a^2} + \beta_1 \frac{\dot{a}}{\dot{b}} + 2\beta_2 \frac{b}{a} \frac{\dot{a}}{\dot{b}} + \beta_3 \frac{b^2}{a^2} \frac{\dot{a}}{\dot{b}} \right), \quad (12)$$

for  $g_{\mu\nu}$  and  $f_{\mu\nu}$ , respectively, where  $H_{(f)} = \dot{a}/a$  ( $\dot{b}/b$ ) is the Hubble constant of  $g_{\mu\nu}$  ( $f_{\mu\nu}$ ), and  $\rho_M = \rho_r + \rho_m$  ( $P_M = P_r + P_m$ ) is the energy density (pressure) of the radiation  $\rho_r$  ( $P_r$ ) and matter  $\rho_m$  ( $P_m$ ).

In this work, we concentrate on the minimum nontrivial case with  $\beta_2 = \beta_3 = \beta_4 = 0$ . Consequently, Eqs. (11) and (12) lead to

$$\frac{b}{a} = \frac{\beta_1}{3H^2}, \quad \text{and} \quad H_b \equiv \frac{H_f}{H} = 1 - 2\frac{\dot{H}}{H^2}. \quad (13)$$

The effective energy density and pressure can be defined from Eqs. (9) and (10)

$$\rho_{DE} = \beta_0 + 3\beta_1 \frac{b}{a} = \rho_{DE}^{(0)} \left( \bar{\beta}_0 + \bar{\beta}_1 \frac{H_0^2}{H^2} \right), \quad (14)$$

$$P_{DE} = -\beta_0 - \beta_1 \left( 2\frac{b}{a} + \frac{\dot{b}}{\dot{a}} \right) = \rho_{DE}^{(0)} \left[ -\bar{\beta}_0 + \bar{\beta}_1 \frac{H_0^2}{H^2} \left( \frac{2\dot{H}}{3H^2} - 1 \right) \right], \quad (15)$$

respectively, which satisfy the continuity equation,  $\rho_{DE} + 3H(\rho_{DE} + P_{DE}) = 0$ , where we have redefined

$$\bar{\beta}_0 = \frac{\beta_0}{\rho_{DE}^{(0)}} \quad \text{and} \quad \bar{\beta}_1 = \frac{\beta_1^2}{H_0^2 \rho_{DE}^{(0)}}, \quad (16)$$

with  $\bar{\beta}_0 + \bar{\beta}_1 = 1$  and  $\rho_{DE}^{(0)}$  the effective dark energy density at present. From Eqs. (14) and (15), one can observe that  $e_0(\mathbb{X})$  with the free parameter  $\beta_0$  in the action plays the role of the cosmological constant, while  $e_1(\mathbb{X})$  with  $\beta_1$  behaves as the inverse quadratic of the Hubble parameter. In Fig. 1, we demonstrate the evolutions of  $\rho_{DE}$  and equation of state (EoS),  $w_{DE} = P_{DE}/\rho_{DE}$ , as functions of the redshift  $z$ . The effective dark energy density grows in the evolution of the universe and reaches a de-Sitter solution,  $H \rightarrow H_{de} = \text{const.}$ , in the future. As a result, it is easy to conclude that  $w_{DE}$  always stays in the phantom phase of  $w_{DE} < -1$ .

### III. LINEAR PERTURBATION THEORY

We start from the generic linear perturbations in the  $k$ -space,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad f_{\mu\nu} = \bar{f}_{\mu\nu} + h_{\mu\nu}^f, \quad (17)$$

where  $\bar{g}_{\mu\nu}$  and  $\bar{h}_{\mu\nu}$  are the background metrics in Eqs. (6) and (7), respectively. In general, there are four scalar modes in each metric, i.e.,

$$h_{00} = -2\Phi, \quad h_{0i} = h_{i0} = 2a^2 \partial_i B, \quad h_{ij} = 2a^2 \left[ \frac{k_i k_j}{2k^2} h + \left( 3\frac{k_i k_j}{k^2} - \delta_{ij} \right) \eta \right], \quad (18)$$

$$h_{00}^f = -2\frac{\dot{b}^2}{\dot{a}^2} \Phi_f, \quad h_{0i}^f = h_{i0}^f = 2b^2 \partial_i B_f, \quad h_{ij}^f = 2b^2 \left[ \frac{k_i k_j}{2k^2} h_f + \left( 3\frac{k_i k_j}{k^2} - \delta_{ij} \right) \eta_f \right], \quad (19)$$

where  $k^2 = k_i k^i$  and  $i = 1, 2, 3$ . Under a gauge transformation,

$$x'^\mu = x^\mu + \epsilon^\mu, \quad (20)$$

with  $\epsilon^\mu$  at the same order of  $h_{\mu\nu}$  and  $h_{\mu\nu}^f$ , we can eliminate two of these scalars. In order to compare the results with the observations, we choose the conventional synchronous gauge with two scalars, ( $h$  and  $\eta$ ), in the ordinary metric [62], and keep four scalars, ( $h_f$ ,  $\eta_f$ ,  $\Phi_f$  and  $B_f$ ), in the new one.

Substituting Eqs. (18) and (19) with  $\Phi = B = 0$  into Eqs. (3) and (4), we have

$$h'' + \left(2 + \frac{H'}{H}\right) h = \frac{\beta_1^2}{6H^4} \left(h - h_f - \frac{3\Phi_f}{H_b}\right) - 3 \frac{(1 + 3w_M) \rho_M}{\rho_M + \rho_{DE}} \delta_M, \quad (21)$$

$$\frac{k^2 \eta'}{a} = \frac{(\bar{\rho}_M + \bar{P}_M) \theta_M}{2H} + \frac{\beta_1^2}{3H^3} \frac{k^2 B_f}{1 + H_b}, \quad (22)$$

and

$$h'_f + 6\eta'_f = \frac{3H_b}{2} (h - h_f) + 2H_b \frac{k^2 \eta_f}{a^2 H^2} - 4 \frac{k^2}{a^2 H^2} \bar{B}_f - \frac{18H_b^3}{1 + H_b} \bar{B}_f, \quad (23)$$

$$\begin{aligned} & \frac{3H_b^2}{1 + H_b} \bar{B}'_f - \left(H_b - 1 - \frac{H'}{H} + \frac{3}{2} H_b^2\right) \frac{\eta'_f}{H_b} \\ &= 3(\eta_f - \eta) - \left(2 + 2\frac{H'}{H} + H_b - \frac{3}{2} H_b^2 + \frac{(2 + H_b) H'_b}{(1 + H_b) H_b}\right) \left(\frac{3H_b^2}{1 + H_b} \bar{B}_f\right), \end{aligned} \quad (24)$$

$$\begin{aligned} h'_f + \frac{12H_b^2}{1 + H_b} \bar{B}'_f &= h' + 12(\eta_f - \eta) + \left(\frac{H'}{H} + 2H_b - 1\right) (h - h_f) + \frac{4k^2 \eta_f}{3a^2 H^2} \left(H_b - 1 - \frac{H'}{H}\right) \\ &- \left(\frac{12H_b^2}{1 + H_b} \bar{B}_f\right) \left[\frac{k^2 H_b}{3a^2 H^2} + 2H_b + \frac{(2 + H_b) H'_b}{(1 + H_b) H_b} + 1 + \frac{H'}{H}\right], \end{aligned} \quad (25)$$

$$\Phi_f = - \left(\frac{3H_b^2}{1 + H_b} \bar{B}_f + \frac{\eta'_f}{H_b}\right), \quad (26)$$

where the prime denotes the derivative of e-folding, i.e. “ $r$ ” =  $d/dN = d/d \ln a$ ,  $\bar{B}_f = a H B_f$ ,  $\tilde{H} \equiv H/H_b - \dot{H}_b/2H_b^3$ ,  $\delta T_0^0 = \delta \rho_M = \rho_M \delta_M$ ,  $\delta T_i^0 = -\delta T_0^i = (\rho_M + P_\ell) v_M^i$ ,  $\delta T_j^i = \delta P_M \delta_j^i$  and  $\theta_M \equiv \partial_i v_M^i$ . In addition, from the conservation equation  $\nabla^\mu T_{\mu\nu}^M = 0$ , one gets

$$\delta'_M = -(1 + w_M) \left(\theta_M + \frac{\dot{h}}{2}\right) - 3 \left(\frac{\delta P_M}{\delta \rho_M} - w_M\right) \delta_M, \quad (27)$$

$$\theta'_M = -(1 - 3w_M) \theta_M - \frac{w'_M}{1 + w_M} \theta + \frac{\delta P_M / \delta \rho_M}{1 + w_M} \frac{k^2 \delta_M}{H}. \quad (28)$$

From Eqs. (21), (22) (27) and (28), it is easy to check that the bimetric massive gravity theory with  $(\bar{\beta}_0, \bar{\beta}_1) = (1, 0)$  is reduced to the  $\Lambda$ CDM limit at not only the background evolution level, but also the linear perturbation one.

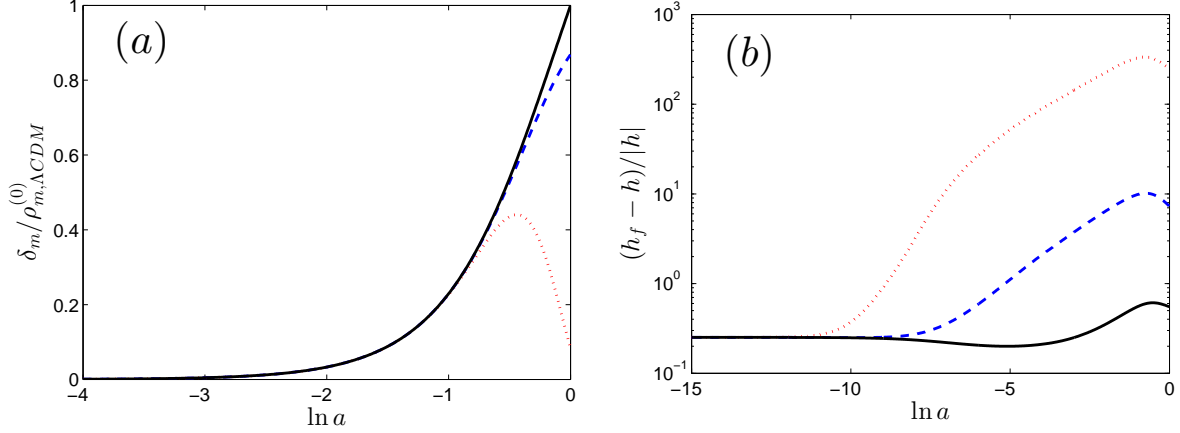


FIG. 2: Evolutions of (a) the matter density perturbation  $\delta_m$ , normalized by  $\delta_m$  in the  $\Lambda$ CDM limit at  $z = 0$  as a function of  $N = \ln a$  with  $(\bar{\beta}_0, \bar{\beta}_1) = (1, 0)$  (black solid line),  $(0.99, 0.01)$  (blue dashed line) and  $(0.9, 0.1)$  (red dotted line), and (b)  $(h_f - h)/|h|$  with  $(\bar{\beta}_0, \bar{\beta}_1) = (0.99, 0.01)$ , and  $k = 10^{-3} [h/\text{Mpc}]$  (black solid line),  $10^{-2} [h/\text{Mpc}]$  (blue dashed line) and  $0.1 [h/\text{Mpc}]$  (red dotted line), respectively, where the boundary conditions are taken to be the same as those in Fig. 1 with  $k = 0.1 [h/\text{Mpc}]$ .

#### IV. COSMOLOGICAL EVOLUTIONS OF MATTER AND SCALAR PERTURBATIONS

The Hubble radius  $d_H \equiv H^{-1}$  expands during the evolution of the universe. More and more density perturbation modes of  $\delta_k$  enter the horizon with the wavenumber  $k$ . By using  $\Omega_m = 0.26$ ,  $\Omega_r = 8.4 \times 10^{-5}$  and  $H_0 = 70 \text{ km/s} \cdot \text{Mpc}$ ,  $\delta_k$  reaches the horizon at the redshifts  $z_k \simeq 32$  and  $8 \times 10^4$  with  $k = 10^{-3}$  and  $0.25 [h/\text{Mpc}]$ , respectively. The matter power spectrum  $P(k) \sim \langle \delta_m^2(k) \rangle$  with the BBKS transfer function can be recovered within 10% accuracy by taking the initial density perturbation to be the scale invariance when all modes are located at the super-horizon scale, i.e.,  $\delta_m = 3\delta_r/4 \propto k^{n_s/2}$ . Therefore, it is reasonable to choose the evolution of  $\delta_m$  with the initial scale factor of  $\ln a_i = -18$ .

By using Eqs. (21) - (28), the evolutions of  $\delta_M$  and  $\theta_M$  as well as the linear perturbation scalars, such as  $h$ ,  $\eta$ ,  $h_f$ ,  $\eta_f$ ,  $B_f$  and  $\Phi_f$ , can be solved. In Fig. 2a, we present the evolution of the matter density perturbation  $\delta_m$  as a function of the e-folding  $N \equiv \ln a$  with  $k = 0.1 [h/\text{Mpc}]$  and  $(\bar{\beta}_0, \bar{\beta}_1) = (1, 0)$  (black solid line),  $(0.99, 0.01)$  (blue dashed line) and  $(0.9, 0.1)$  (red dotted line). When  $h_f + 3\Phi_f/H_b > h$  in Eq. (21), the growth of the matter

density perturbation is smaller than that in the  $\Lambda$ CDM case ( $\bar{\beta}_1 = 0$ ). As a result,  $\delta_m$  in the bimetric massive gravity theory is suppressed compared to that in the  $\Lambda$ CDM model. In Fig. 2b, the difference of the scalars,  $(h_f - h)/|h|$ , is represented as a function of  $N$  with  $(\bar{\beta}_0, \bar{\beta}_1) = (0.99, 0.01)$  with  $k = 10^{-3}[h/Mpc]$  (black solid line),  $10^{-2}[h/Mpc]$  (blue dashed line) and  $0.1[h/Mpc]$  (red dotted line), respectively. At the super-horizon scale, we see that  $h_f \gtrsim h$  in the evolution of the universe, while the detailed calculations in Eqs. (A6) and (A7) show that  $0.75h \simeq h_f < 0$  in both radiation and matter dominated epochs. When the  $k$ -mode is in the horizon,  $h_f$  sharply increases with  $h_f \propto a^{\bar{\lambda}+c}$  and  $\eta_f \propto a^{\bar{\lambda}}$ , where  $c = 2$  and 1 at the radiation and matter dominated epochs, respectively. In Eqs. (A10) and (A11), we find that  $\bar{\lambda} \sim 0.5$  before the dark energy dominated era. The larger  $k$  is, the earlier the mode enters the horizon, resulting in a more significant enhancement of  $h_f$ . Although the bimetric massive gravity theory restrains the growth of the matter density perturbation, the suppression effect of  $\delta_m$  would be negligible by  $H^{-4}$  in the early time (see also Eq. (21)) until

$$\left| \frac{\beta_1^2 h_f}{H^4} \right| > \delta_M. \quad (29)$$

Clearly, such a suppression depends on both the wavenumber  $k$  and model parameter  $\beta_1$ .

## V. GHOST AND INSTABILITY PROBLEMS AND MATTER POWER SPECTRUM

It is known that the bimetric massive gravity theory suffers from the Higuchi ghost and instability problems [23–40] due to the negative squared mass of graviton and the divergence under the linear density perturbation, respectively. In order to avoid the ghost problem, the condition of  $(b/a)' \geq 0$  is required, which can be derived from Ref. [41]. From Eq. (13), we see that  $(b/a)' \propto -\dot{H}/H^4$ , which is larger than or equal to zero with the equal sign at  $H = H_{de}$  in the late-time dark energy dominated epoch. Clearly, our minimum case of the bimetric massive gravity theory is ghost-free in the cosmological evolution. On the other hand, the stability condition in Ref. [41] is broken in the matter dominated epoch with  $\beta_i = 0$  ( $i = 2, 3$  and 4). As shown in Fig. 2b, our numerical result confirms such a divergence property on the growth of the linear scalar perturbation for the new metric,  $h_f$ , in the sub-horizon scale. The sharply increasing  $h_f$  leads to the suppression of the cosmological observables,  $\delta_m$ ,  $h$



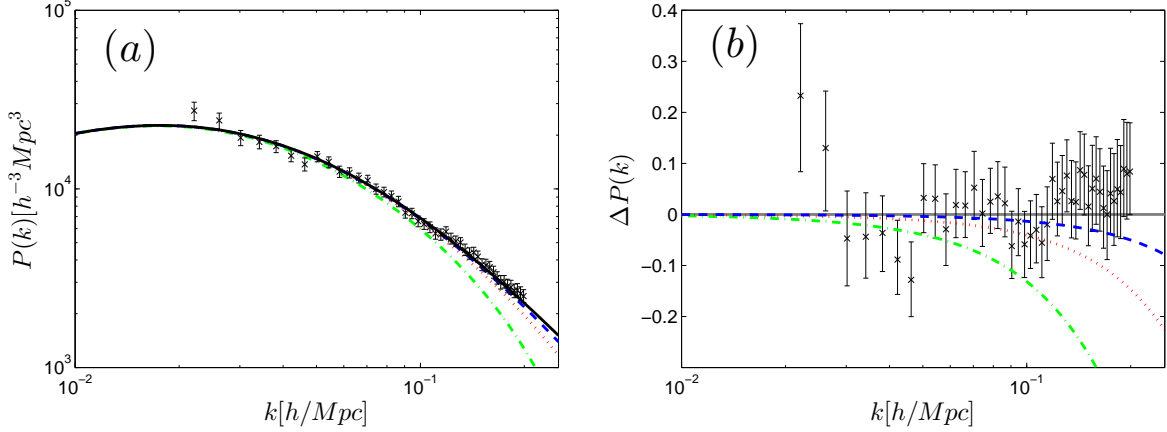


FIG. 3: (a) The matter power spectrum  $P(k)$  and (b)  $\Delta P(k) = (P - P_{\Lambda\text{CDM}})/P_{\Lambda\text{CDM}}$  as functions of the wavenumber  $k$  with  $\sum m_\nu = 0.2$  eV and  $\bar{\beta}_1 = 0$  (black solid line),  $10^{-3}$  (blue dashed line),  $3 \times 10^{-3}$  (red dotted line) and  $10^{-2}$  (green dash-dotted line), together with the data points come from the SDSS LRG DR7. Note that  $P_{\Lambda\text{CDM}}$  corresponds to the case with  $\bar{\beta}_1 = 0$ , and the boundary conditions are taken to be the same as those in Fig. 1.

and  $\eta$ . Fortunately, the suppression strength, which can be estimated from Eqs. (21) and (22), is proportional to  $\beta_1^2 \sim \bar{\beta}_1$ . As a result, when  $\bar{\beta}_1$  is small enough, the deviation of  $\delta_m$  in the bimetric massive gravity theory from that in the  $\Lambda\text{CDM}$  model can be limited. By comparing the deviation of the matter power spectrum,  $P(k) \propto |\delta_m|^2$ , to the large scale structure observations, we can give an upper bound for  $\bar{\beta}_1$ . In Fig. 3, we illustrate  $P(k)$  as a function of  $k$  with a massive neutrino of  $m_\nu = 0.2$  eV and  $\bar{\beta}_1 = 0$  (black solid line),  $10^{-3}$  (blue dashed line),  $3 \times 10^{-3}$  (red dotted line) and  $10^{-2}$  (green dash-dotted line), respectively, where the data points come from the SDSS LRG DR7 [63]. As we can see, the suppression of  $P(k)$  is quite strong, which allows us to make a statement that the observational data constrain the allowed window to be  $\bar{\beta}_1 \lesssim \mathcal{O}(10^{-3})$  in the bimetric massive gravity theory.

Additionally, one has

$$\frac{(b/a)''}{b/a} = 9(1 + w_t)^2 + 3w_t' \lesssim \left( \frac{(b/a)'}{b/a} \right)^2 = 9(1 + w_t)^2. \quad (30)$$

which satisfies the stability condition in Ref. [41] in the dark energy dominated era, where we have used  $w_t = (P_M + P_{DE})/(\rho_M + \rho_{DE})$  and  $w_t' \lesssim 0$ . Consequently, although the observables are unstable under the linear scalar perturbations in the matter dominated epoch, they become stabilized again when dark energy dominates the universe.

## VI. CONCLUSIONS

We have studied the matter density perturbation  $\delta_m$  and matter power spectrum  $P(k)$  in the bimetric massive gravity theory for the minimal scenario with  $\beta_2 = \beta_3 = \beta_4 = 0$ . In this approach,  $\beta_0 e_0(\mathbb{X})$  in the action plays the role of the cosmological constant in both background and linear perturbation levels, while  $\beta_1 e_n(\mathbb{X})$  behaves as an inverse quadratic of  $H$ . We have shown that the effective dark energy EoS in the theory is always at the phantom phase, i.e.,  $w_{DE} < -1$ .

By taking the synchronous gauge in  $g_{\mu\nu}$ , we have calculated the evolution of the matter density perturbation in our minimal case of the bimetric massive gravity theory. Through the analytical discussion, the growth of the scalars in the new metric is independent on the choices of  $\beta_0$  and  $\beta_1$ . At the scale outside the horizon,  $k^2/a^2 \ll H^2$ , we have the relations,  $h_f \sim h \gg \eta_f \sim \Phi_f \gg \bar{B}_f$ . When the scale enters the horizon,  $k^2/a^2 \gg H^2$ , the growths of  $\eta_f$ ,  $\Phi_f$  and  $\bar{B}_f$  are suspended, and  $h_f$  sharply increases, causing the suppression of the matter power spectrum  $P(k)$ . This effect depends on both the scale  $k$  and parameter set  $(\beta_0, \beta_1)$ .

Even if the linear scalar perturbations in the new metric diverge in the matter dominated epoch,  $\delta_m$  can still fit to the cosmological observations with a small enough  $\bar{\beta}_1$ . Comparing our numerical results of  $P(k)$  with the data, we conclude that the allowed model parameter window is  $\bar{\beta}_1 \lesssim \mathcal{O}(10^{-3})$ .

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### Appendix A: Growths of Scalar Modes in Bimetric Massive Gravity

We present the detailed calculation of the evolution equations for the scalar perturbations in the bimetric massive gravity theory, including two scalar perturbations,  $h$  and  $\eta$ , in the

ordinary metric and four scalar perturbations,  $h_f$ ,  $\eta_f$ ,  $B_f$  and  $\Phi_f$ , in the new metric. From Eqs. (23) - (25), we can observe that the evolutions of the scalar perturbations in the new metric is independent on the model parameter  $\beta_1$ , so that  $\beta_1 \rightarrow 0$  is taken in our discussion of the asymptotic behavior in the radiation and matter dominated epochs in this section.

### 1. Scalar perturbations at the super-horizon scale, $k^2 \ll a^2 H^2$

At the super-horizon scale, Eqs. (23) - (25) are reduced to

$$h'_f + 6\eta'_f \simeq \frac{3(4 + 3w_M)}{2} (h - h_f) - \frac{18(4 + 3w_M)^3}{5 + 3w_M} \bar{B}_f, \quad (\text{A1})$$

$$\begin{aligned} \frac{3(4 + 3w_M)^2}{5 + 3w_M} \bar{B}'_f - \frac{(57 + 81w_M + 27w_M^2)}{2(4 + 3w_M)} \eta'_f &\simeq 3(\eta_f - \eta) \\ &+ \frac{9(4 + 3w_M)^2(14 + 24w_M + 9w_M^2)}{2(5 + 3w_M)} \bar{B}_f, \end{aligned} \quad (\text{A2})$$

$$h'_f + \frac{12(4 + 3w_M)^2}{5 + 3w_M} \bar{B}'_f \simeq h' + 12(\eta_f - \eta) + \frac{(11 + 9w_M)}{2} (h - h_f) - 18(4 + 3w_M)^2 B_f, \quad (\text{A3})$$

where we have used  $w_M = \text{const.}$ ,  $\dot{H}/H^2 = -3(1 + w_M)/2$  and  $H_b = 4 + 3w_M$ , and redefined  $\bar{B}_f = aHB_f$ , while the prime denotes the derivative of e-folding, i.e. “ $r$ ” =  $d/d \ln a$ .

It is known that  $\delta_M$  and  $h \propto a^\lambda$  in the radiation ( $w_M = 1/3$ ,  $\lambda = 2$ ) and matter ( $w_M = 0$ ,  $\lambda = 1$ ) dominated epochs, respectively. Substituting the relations,

$$h_f \propto a^{\lambda_h}, \quad \eta_f \propto a^{\lambda_\eta} \quad \text{and} \quad B_f \propto a^{\lambda_B}, \quad (\text{A4})$$

into Eqs. (23) - (25), the only possible solution is

$$\lambda_h = \lambda_\eta = \lambda_B = \lambda. \quad (\text{A5})$$

Combining the equations above, the scalar perturbations are found to be explicitly functions of  $h$  and  $\eta$ , given by

$$h_f \sim 0.748h, \quad \eta_f \sim -0.057h + 0.405\eta \quad \text{and} \quad B_f \sim 2.87 \times 10^{-3}h - 0.013\eta \quad (\text{A6})$$

and

$$h_f \sim 0.822h, \quad \eta_f \sim -0.040h + 0.585\eta \quad \text{and} \quad B_f \sim 2.08 \times 10^{-3}h - 0.015\eta \quad (\text{A7})$$

in the radiation and matter dominated eras, respectively.

## 2. Scalar perturbations at the sub-horizon scale, $k^2 \gg a^2 H^2$

When the  $k$ -mode enters the horizon, the  $k^2/a^2 H^2$  terms play the most important role in the cosmological evolution. Substituting Eq. (A4) into Eqs. (23) - (25), the growth powers are deduced as

$$\lambda = \lambda_\eta = \lambda_B = \lambda_h - c, \quad (\text{A8})$$

where  $c = 2$  and  $1$  for the radiation ( $w_M = 1/3$ ) and matter ( $w_M = 0$ ) dominated eras, respectively. Thus, we have

$$h_f \sim \frac{k^2 \eta_f}{a^2 H^2} \quad \text{and} \quad \frac{k^2 \bar{B}_f}{a^2 H^2}, \quad (\text{A9})$$

which allow us to take that  $\bar{B}_f = \mathcal{C} \eta_f \ll h_f$  at the scale deep inside the horizon. Finally, the values of  $\mathcal{C}$  and  $\lambda$  can be solved by substituting Eq. (A8) into Eqs. (23) - (25), given by

$$w_M = \frac{1}{3} : \quad \lambda \simeq 0.577, \quad \text{and} \quad \mathcal{C} \simeq -0.019, \quad (\text{A10})$$

$$w_M = 0 : \quad \lambda \simeq 0.432, \quad \text{and} \quad \mathcal{C} \simeq -0.031, \quad (\text{A11})$$

which match our numerical calculations of  $\lambda \sim 0.58$  for  $w_M = 1/3$  and  $\lambda \sim 0.43$  for  $w_M = 0$ . These results show a clear behavior that the growth of  $h_f$  is enhanced with  $h_f \propto a^{\lambda+c}$ , but those of  $\eta_f$  and  $B_f$  are suppressed with  $|\eta_f| \gg |\bar{B}_f| \propto a^\lambda$  in the matter dominated era. As a result, we conclude that

$$h_f \sim h \quad (\text{A12})$$

at the super-horizon scale, and

$$|h_f| \gg |h| \gg |\eta_f| > |\bar{B}_f| \quad (\text{A13})$$

at the scale deep inside the horizon.

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